Name: Solutions
Start Time:
End Time:
Date:

Math 260
Quiz 4 (45 min)

1. (1 point) If $A = \begin{bmatrix} 1 & 7 \\ 3 & -6 \\ 0 & 2 \end{bmatrix}$, find A^T

$$A^T = \begin{bmatrix} 1 & 3 & 0 \\ 7 & -6 & 2 \end{bmatrix}$$

2. (2 points) If $A = \begin{bmatrix} 7 & -2 \\ 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$, find AB.

$$AB = \begin{bmatrix} 7 & -2 \\ 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (7)(0)+(-2)(1) & (7)(3)+(-2)(-1) & (7)(2)+(-2)(2) \\ (1)(0)+(4)(1) & (1)(3)+(4)(-1) & (1)(2)+(4)(2) \\ (2)(0)+(3)(1) & (2)(3)+(3)(-1) & (2)(2)+(3)(2) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -2 & 23 & 10 \\ 4 & -1 & 10 \\ 3 & 3 & 10 \end{bmatrix}}$$

3. (4 points) When writing the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 10 \\ 5x_1 + x_2 - 2x_3 &= 1 \\ x_1 - 2x_2 &= 1 \\ -2x_1 + 7x_3 &= 12 \end{aligned}$$

as the matrix equation $A\vec{x} = \vec{b}$, find…

a) A

b) $\vec{x}$ (don't solve for $\vec{x}$ here)

c) $\vec{b}$

$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & -2 \\ 1 & -2 & 0 \\ -2 & 0 & 7 \end{bmatrix}$ $\quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\quad \vec{b} = \begin{bmatrix} 10 \\ 1 \\ 1 \\ 12 \end{bmatrix}$

d) Row reduce an augmented matrix to solve for $\vec{x}$ (calculator OK)

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 10 \\ 5 & 1 & -2 & 1 \\ 1 & -2 & 0 & 1 \\ -2 & 0 & 7 & 12 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 1 \\ x_2 = 0 \\ x_3 = 2 \end{array} \Rightarrow \boxed{\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}}$$

4. (3 points) Prove: If A and B are $m \times n$ matrices, then $(A+B)^T = A^T + B^T$.

proof: Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be $m \times n$ matrices.

$\boxed{\text{sizes}}$ left/ Since A and B are $m\times n$, $A+B$ is $m\times n$, so $(A+B)^T$ is $n \times m$

Right/ A $m\times n \Rightarrow A^T$ $n \times m$. Also B $m\times n \Rightarrow B^T$ $n\times m$

So $A^T + B^T$ is also $n \times m$ ← same

$\boxed{\text{corresponding entries equal?}}$

Left: $(A+B)^T = ([a_{ij}] + [b_{ij}])^T = [a_{ij} + b_{ij}]^T = [a_{ji} + b_{ji}]$

Right $= A^T + B^T = [a_{ij}]^T + [b_{ij}]^T = [a_{ji}] + [b_{ji}]$
$= [a_{ji} + b_{ji}]$.

So the (i,j)-th entry of both sides is $a_{ji} + b_{ji} \Rightarrow (A+B)^T = A^T + B^T$.

Extra Credit

1. (2 points) Prove: If $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x} = \vec{0}$, and a and b are scalars, then $a\vec{x}_1 + b\vec{x}_2$ is also a solution $A\vec{x} = \vec{0}$.

proof: Suppose $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x} = \vec{0}$ and let a and b be scalars. Then $A\vec{x}_1 = \vec{0}$ and $A\vec{x}_2 = \vec{0}$.

Then $A(a\vec{x}_1 + b\vec{x}_2) = A(a\vec{x}_1) + A(b\vec{x}_2) = a(A\vec{x}_1) + b(A\vec{x}_2)$

$= a\vec{0} + b\vec{0} = \vec{0} + \vec{0} = \vec{0}$,

So $a\vec{x}_1 + b\vec{x}_2$ is also a solution to $A\vec{x} = \vec{0}$.

2. (2 points) Prove or disprove: If A and B are 2×2 matrices, then $AB = BA$.

Counterexample $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$.

Then $AB = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$

and $BA = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Not same

So $AB \neq BA$.